Testing of Hypothesis in MLRM

Y	X_1	X_2
100	5	1000
75	7	600
80	6	1200
70	6	500
50	8	300
65	7	400
90	5	1300
100	4	1100
110	3	1300
60	9	300

Y = q^n demanded of a certain commodity

X_1 = price of the commodity

X_2 = consumer income

i) Test $H_0: \beta_0 = 0$ against $H_1: \beta_0 \neq 0$ and find 95% and 99% confidence interval for β_0

ii) Similarly for β_1 and β_2.

$n=10$, $\Sigma Y_i = 800$, $\Sigma X_{1i} = 60$, $\Sigma X_{2i} = 8000$, $\overline{Y} = 80$, $\overline{X}_1 = 6$, $\overline{X}_2 = 80$

$\Sigma y_i = 0$ $\Sigma x_{1i} = 0$ $\Sigma x_{2i} = 0$ $\Sigma y_i^2 = 3450$ $\Sigma x_{1i}^2 = 30$

$\Sigma x_{2i}^2 = 1580,000$ $\Sigma x_{1i} y_i = 300$, $\Sigma x_{2i} y_i = 65000$ $\Sigma x_{1i} x_{2i} = 5900$

$\hat{\beta}_1 = -7.19$ $\hat{\beta}_2 = 0.0143$ $\hat{\beta}_0 = 111.70$

$$R^2 = \frac{ESS}{TSS} = \frac{\Sigma \hat{y}_i^2}{\Sigma y_i^2} = \frac{\hat{\beta}_1 \Sigma x_{1i} y_i + \hat{\beta}_2 \Sigma x_{2i} y_i}{\Sigma y_i^2}$$

$$= 0.894$$

$$\overline{R}^2 = 1 - (1-R^2)\frac{n-1}{n-3} \quad \text{Here } n = 10$$

$$= 0.8637$$

$SE(\hat{\beta}_1) = 2.5592$ $SE(\hat{\beta}_2) = 0.0111$ $SE(\hat{\beta}_0) = 23.570$

i) $H_0: \beta_0 = 0$

$H_1: \beta_0 \neq 0$

We know,

$$t = \frac{\hat{\beta}_0}{SE(\hat{\beta}_0)} = \frac{111.70}{23.570} = 4.739$$

The null hypothesis $H_0: \beta_0 = 0$ will be accepted if for the given sample $-t_{\alpha/2, n-3} \leq t \leq t_{\alpha/2, n-3}$ and will be rejected otherwise. [Note: d.f. $= n-3$ because in MLRM d.f. is given by $n-K$ and here $K = 3$ (no. of parameters i.e. $\beta_0, \beta_1, \beta_2$)]

When $\alpha = 0.05$, $t_{\alpha/2, n-3} = t_{0.025, (10-3)} = t_{0.025, 7} = 2.365$ (table value given)

Thus we see that observed t value 4.739 doesn't lie in the interval -2.365 and 2.365; so we can reject the null hypothesis $H_0: \beta_0 = 0$ and accept the alternate hypothesis $H_1: \beta_0 \neq 0$ at 5% level of significance.

Similarly when $\alpha = 0.01$, $T_{\alpha/2, n-3} = T_{0.005, 7} = 3.499$.. Here also we see that observed T value 4.739 doesnt lie in the interval -3.499 and 3.499 so we reject the null hypothesis and accept the alternate hypothesis at 1% level of significance.

We know that $100(1-\alpha)\%$ confidence interval for β_0 is given by ______

$$P\left[-T_{\alpha/2, n-3} \leqslant T \leqslant T_{\alpha/2, n-3}\right] = 1-\alpha$$

or, $$P\left[-T_{\alpha/2, n-3} \leqslant \frac{\hat{\beta}_0 - \beta_0}{SE(\hat{\beta}_0)} \leqslant T_{\alpha/2, n-3}\right] = 1-\alpha$$

or $$P\left[\hat{\beta}_0 - T_{\alpha/2, n-3} \cdot SE(\hat{\beta}_0) \leqslant \beta_0 \leqslant \hat{\beta}_0 + SE(\hat{\beta}_0) \cdot T_{\alpha/2, n-3}\right] = 1-\alpha$$

When $\alpha = 0.05$,

$$P\left[\hat{\beta}_0 - T_{0.025, 7}(23.570) \leqslant \beta_0 \leqslant \hat{\beta}_0 + T_{0.025, 7}(23.570)\right] = 1-0.05$$

or, $$P\left[111.70 - (2.365 \times 27.570) \leqslant \beta_0 \leqslant 111.70 + (2.365 \times 23.570)\right] = 0.95$$

or, $$P\left[55.957 \leqslant \beta_0 \leqslant 166.743\right] = 0.95$$

$\therefore$ 95% confidence interval for β_0 are 55.957 and 166.743.

Similarly when $\alpha = 0.01$ then $100(1-\alpha)\% = 99\%$.

So, 99% confidence interval of β_0 would be,

$$\hat{\beta}_0 \pm T_{\alpha/2, n-3} SE(\hat{\beta}_0)$$

or, $$\hat{\beta}_0 \pm T_{0.005, 7} SE(\hat{\beta}_0)$$

or, $$111.70 \pm 3.499 \times 23.570$$

or, $$111.70 \pm 82.4717$$

i.e. 29.2286 and 194.1714

For, β_1

observed $T = -2.8094$

when $\alpha = 0.05$, $T_{0.025,7} = 2.365$ (table value)

$\Rightarrow$ observed T doesn't lie in the interval -2.365 and 2.365 so we reject the null hypothesis at 5% level of significance.

when $\alpha = 0.01$, $T_{0.025,7} = 3.499$

$\Rightarrow$ H_0 is accepted at 1% level of significance.

$\Rightarrow$ 95% confidence interval for β_1 is -13.245 and -1.135

$\Rightarrow$ 99% " " " " " -16.1446 and 1.7646

For β_2, Tabulated value of $T_{0.025,7} = 2.365$ (at $\alpha = 0.05$)

and $T_{0.005,7} = 3.499$ (at $\alpha = 0.01$)

Home Assignment: Complete the hypothesis testing for β_2 and find the 95% and 99% confidence interval.